Effectiveness of Mixing Tanks in Smoothing Cyclic Fluctuations

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It is the purpose of this note to show how problems of the type discussed by Gutoff (*1*) can be handled more effectively with standard transfer function methods and block diagrams (2).

The mixer being considered is shown in Figure 1. The feed rate is F and the agitator pumps liquid at the rate P through the draught tube. Gutoff considered the case of a pure time delay in the draught tube and no delay in the recirculation of liquid through the vessel. In order to generalize the expressions Gutoff's nomenclature can be extended to include $C_m(t)$, the concentration immediately after mixing of feed and recycle, and $C_r(t)$, the concentration of the recycle immediately before mixing.

The equation for the mixing process is

$$C_m(t) = \frac{F}{P} C_i(t) + \frac{(P-F)}{P} C_r(t) \tag{1}$$

and for the time delay

$$C_o(t) = C_m(t - T_1) \tag{2}$$

Furthermore in this case

$$C_r(t) = C_o(t) \tag{3}$$

Transforming these equations one gets

$$C_m(s) = \frac{F}{P} C_i(s) + \frac{(P-F)}{P} C_r(s) \tag{4}$$

and

$$C_r(s) = C_o(s) = e^{-T_1 s} C_m(s) \tag{5}$$

These equations are represented in block diagram form in Figure 2.

$\frac{C_o(s)}{C_m(s)} = e^{-T_1 s}$ is the transfer function for a pure time delay. The combination of feed and recycle concentration signals is represented by the summing point at the left.

The transfer function for the system as a whole is, by inspection

$$\frac{C_o(s)}{C_i(s)} = \frac{e^{-T_1 s} \frac{F}{P}}{1 - e^{-T_1 s}\left(1 - \frac{F}{P}\right)}$$

The ratio of standard deviations for output to input concentration fluctuations is, in the case of sinusoidal oscillations, the same as the ratio of the amplitudes of the oscillations, which is termed the "dynamic gain" of the system. This is obtained directly from the transfer function by substituting $i\omega$ for s and by determining the modulus of the resulting complex number.

In this case the steps give

$$\left|\frac{C_o(i\omega)}{C_i(i\omega)}\right| = \left|\frac{e^{-i\omega T_1} \frac{F}{P}}{1 - e^{i\omega T_1}\left(1 - \frac{F}{P}\right)}\right|$$

$$= \left|\frac{\frac{F}{P}\left(\text{Cos}\,(-\omega T_1) + i\,\text{Sin}\,(-\omega T_1)\right)}{1 - \left(1 - \frac{F}{P}\right)\left(\text{Cos}\,(-\omega T_1) + i\,\text{Sin}\,(-\omega T_1)\right)}\right| \tag{6}$$

This reduces by using standard methods and by writing $F/P = T/H$ and $\omega = 2\pi/\lambda$ to Gutoff's equation (10).

The advantage of the transfer function method of analysis is the ease with which alternative models may be considered. For example if the recycle stream is considered to be perfectly mixed in the vessel, then the relationship between $C_r(t)$ and $C_o(t)$ is given by the transfer function

$$\frac{C_r(s)}{C_o(s)} = \frac{1}{1 + T_2 s} \tag{7}$$

The block diagram for this case is a simple extension of Figure 2 and is shown in Figure 3.

The transfer function for the system is now

$$\frac{C_o(s)}{C_i(s)} = \frac{e^{-T_1 s} \frac{F}{P}}{1 - e^{-T_1 s}\left(1 - \frac{F}{P}\right)\left(\frac{1}{1 + T_2 s}\right)} \tag{8}$$

and the dynamic gain is given by

$$\left|\frac{C_o(i\omega)}{C_i(i\omega)}\right| = \frac{(1 + k^2 \omega^2 T_1^2)^{\frac{1}{2}}}{\left(1 + 2\frac{P}{F}\left(\frac{P}{F} - 1\right)(1 - \text{Cos}\,\omega T_1 + k\omega T_1 \,\text{Sin}\,\omega T_1) + k^2 \frac{P^2}{F^2}\omega^2 T_1^2\right)^{\frac{1}{2}}}$$

where $k = \frac{T_2}{T_1}$

This expression for dynamic gain is plotted vs. ωT_1 in Figure 4 for $P/F = 2$ and for various values of k.

For k equal to zero the results are the same as for Gutoff's model. As can be seen from Figure 4 the gain does not fall appreciably below 0.5, which means that amplitude of concentration fluctuations could not be reduced by more than 50%.

The foregoing manipulations of block diagrams and transfer functions illustrate the ease with which variants of proposed models may be devised and tested. This type of procedure might be termed "mathematical experimentation" and is easily carried out.

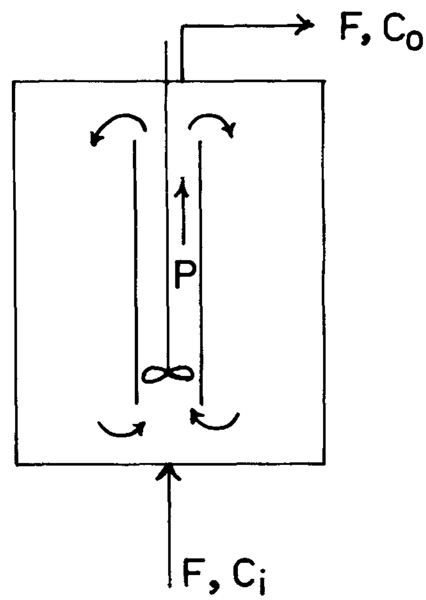

Fig. 1. Mixer with draught tube.

A more pressing need is for comparison of experimental results on mixers with the results of the mathematical experiments in order to determine

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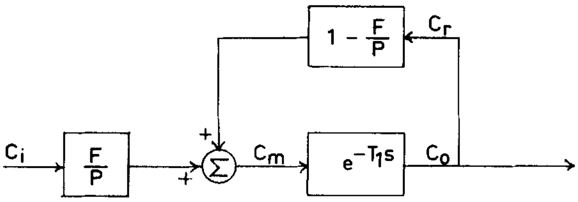

Fig. 2. Block diagram representation of mixer with zero recirculation delay.

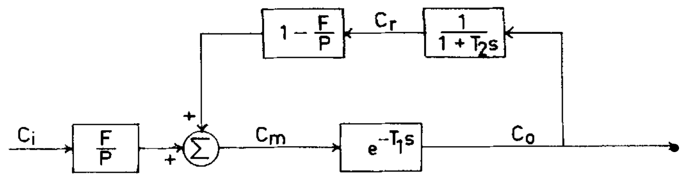

Fig. 3. Block diagram representation of mixer with a perfectly mixed recycle stream.

the most useful models and methods of relating model parameters to the physical dimensions of the equipment.

NOTATION

$C_i(t)$ = input concentration or other additive intensive property
$C_m(t)$ = concentration after mixing
$C_o(t)$ = output concentration
$C_r(t)$ = recycle concentration before mixing
$C(s)$ = Laplace transform of $C(t) = \int_0^\infty e^{-st}\, C(t)dt$
i = $(-1)^{1/2}$
k = T_2/T_1
F = flow rate in and out of the mixing tank, lb/min.
P = agitator pumping rate
T_1 = time delay in the draught tube, min.
T_2 = time constant of the mixing tank apart from the draught tube, min.
ω = frequency of sinusoidal oscillation radians/min.
λ = period of sinusoidal oscillation, min.

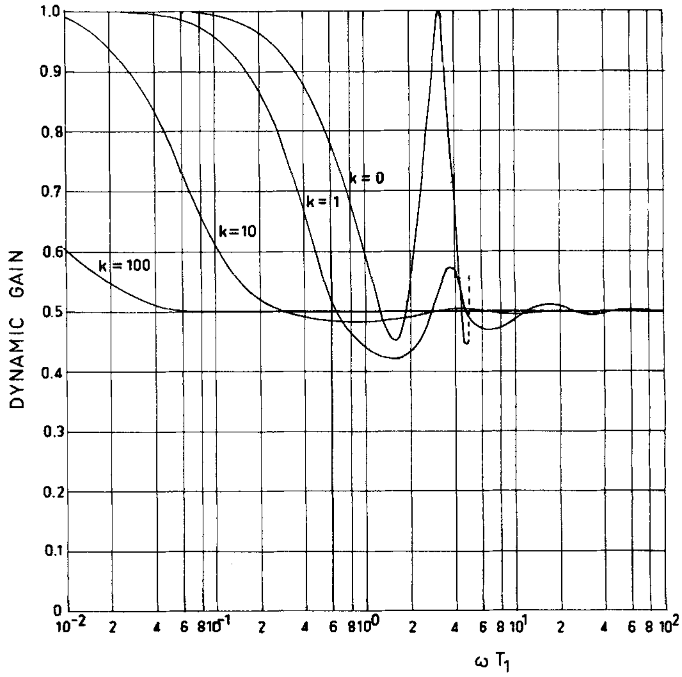

Fig. 4. Dynamic gain of mixer with perfectly mixed recycle stream.

LITERATURE CITED

1. Gutoff, E. B., *A.I.Ch.E. Journal*, 6, 347 (1960).
2. Ceaglske, N. A., "Automatic Process Control for Chemical Engineers," Wiley, New York (1956).

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